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D1-brane with overcritical electric field in AdS_3 and S-brane

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Abstract: We study aspects of Dirichlet S-branes, which are defined as Dirichlet boundary condition on a time like embedding of open strings, in general backgrounds. By applying T-duality along an isometry of the unphysical dS_2-branes in NS-NS supported AdS_3-background, we find S0-brane. We also study the time dependent tachyon condensation on the unstable Dp-brane and interpret the singular solutions as lower dimensional S(p-1)-brane that couples to real Ramond-Ramond fields while to imaginary NS-NS modes.

Keywords: Tachyon Condensation, D-branes.

Contents

1. Introduction

Space-like branes or S-branes [1] are fascinating objects in string theory. They are defined as a kind of topological defects localized on a space-like hypersurface, and hence can only exist for 'moment' in time. The rolling of the open string tachyon on an unstable D-brane (for a review see [2] and references therein) namely the decaying branes to the closed string vacuum in some cases can be considered as an array of Dirichlet S-branes in imaginary time [3]. In general S-branes can also be viewed as the time-dependent homogeneous solutions in string theory or in supergravity, localized in a given instant of time. They have been very useful in understanding cosmological applications of string theory.

Dirichlet S-branes are also obtained by imposing a Dirichlet condition on the time like coordinate of the open strings [4]. Under T-duality along a transverse spatial direction the S-branes are shown to be T-dual to the D-branes with overcritical electric field. It was further observed that unlike the D-branes, in the first quantization of the open string between a pair of S-branes, there are only a finite number of physical states that increases when they gets separated with time. In general S-brane solutions in the type-II string theory can be obtained by analytically continuing the usual D-brane boundary states, but one has to keep in mind that it radiates the Ramond-Ramond field with wrong reality property. In other words one can have a S-brane with real R-R charge, but then it won't be a solution of type-II theories rather its existence can be predicted in II* theory. Further it was shown that the generic S-brane configurations should decay into a bunch of D-branes (or brane-anti-brane pairs).

D-branes in the Anti-de Sitter backgrounds have been studied by various authors in the past by using various techniques, see for example ([5]–[15]). String theory on the SL(2,R) and its discrete orbifolds have shed new light in the conjectured AdS/CFT duality. The

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corresponding target space geometry is AdS_3 supported by NS-NS three-form flux. D-branes in this background has been considered in the past. In [5], it was shown that the dS_2-branes in AdS_3 are unphysical due to the presence of overcritical electric field. So it is tempting to examine the behavior of these unphysical D-branes in the T-dual background in the light of [4]. We address this question in this paper. In doing so, what we achieve is the following. First of all we are able to find a physical interpretation of the unphysical solutions with imaginary electric flux etc as in the T-dual picture the S0-brane that arises from the time dependent tachyon condensation on unstable D1-brane. Second, the previously found unphysical solutions correspond, in fact, to perfect and acceptable solutions in string theory (even if the initial configurations of tachyon that corresponds to S-brane have to be fine tuned) since they arise from the open string tachyon condensation.

The rest of the paper is organized as follows. In section 2 we try to spell out some properties of the Anti-de Sitter D-branes and show the unphysicalness of the dS_2-branes in AdS_3. In section 3, we apply T-duality along one of the symmetry directions, and interpret the solution that can be seen as D0-brane moving along that particular direction. We find out the equation of motion for the dynamical variable for the later comparison with the S0-branes. In section 4, we study the time dependent tachyon condensation on the unstable D-brane and found out the signature of the $S(p-1)$ branes. In fact, we found that the dynamics of the kink is governed by the equations of motion that arise from the S-brane effective action in given background [1]. We further analyze the properties of energy-momentum tensor derived from such DBI action. The main result of this analysis is the fact that the singular time dependent tachyon condensation on an unstable Dp-brane leads to the emergence of the object whose equations of motion arises from the action that can be interpreted as S(p-1)-brane with imaginary tension (in other words, it couples to imaginary NS-NS modes) and with real charge with respect to Ramond-Ramond fields. This is equivalent to the analysis performed in [4] where this kind of Sp-branes was named as $S^{-}p$-brane [2]. Then we apply this general procedure in section 5 to find out the S0-brane equations of motion that resembles with that of the D-brane in the dual background. Finally in section 6, we present our conclusions.

2. D-branes with overcritical electric fields and emerging S-branes

In this section we study the properties of the D-branes with overcritical electric fields. Let us begin with the $AdS_3 \times S^3$ metric in global coordinates:

$$ds^2 = L^2\left[-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\theta_1^2\right] + L^2\left[d\theta^2 + \cosh^2\theta d\tilde{\psi}^2 + \sin^2\theta d\theta_2^2\right] \tag{2.1}$$

[1] S-brane action was also studied in [16 – 19].

[2] It is necessary to mention one important subtlety considering our results and the work [4]. It was argued there that the $S^{-}p$-brane should contain open string tachyon in its world volume theory. Unfortunately using effective field theory description performed below we are not be able to find the evidence for the existence of this tachyon. It is of course possible that more general ansatz for fluctuations around the time dependent tachyon solution of the non-BPS Dp-brane world volume theory will contain in its spectrum a tachyonic mode. We hope to return to this problem in future.

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supported by the Neveu-Schwarz three-form field

$$H = dB = L^2 \sinh(2\rho) d\rho \wedge d\theta_1 \wedge dt \,, B = L^2 \sinh^2 \rho d\theta_1 \wedge dt \,. \tag{2.2}$$

Let us consider the D1-brane in the above background with the DBI action

$$S = -\tau_1 \int d^2\xi e^{-\Phi} \sqrt{-\det \mathbf{A}} \,, \tag{2.3}$$

with

$$\mathbf{A}_{\mu\nu} = g_{MN} \partial_\mu X^M \partial_\nu X^N + b_{MN} \partial_\mu X^M \partial_\nu X^N + (2\pi\alpha')(\partial_\mu A_\nu - \partial_\nu A_\mu) \,, \tag{2.4}$$

where Dp-brane tension is equal to

$$\tau_p = \frac{1}{(2\pi)^{\frac{(p-1)}{2}} (2\pi\alpha')^{\frac{p+1}{2}}} \,, \tag{2.5}$$

$X^M \,, M = 0, \dots, 9$ label the position of D1-brane, $g_{MN} \,, b_{MN}$ are background metric and NS-NS two form field respectively and $A_\mu \,, \mu = 0, 1$ is worldvolume gauge field. Now the equations of motion for X^K derived from the action (2.3) take the form

$$\begin{aligned} &\partial_K [\tau_p e^{-\Phi}] \sqrt{-\det \mathbf{A}} + \frac{\tau_p e^{-\Phi}}{2} \left[\partial_K g_{MN} + \partial_K b_{MN}\right] \partial_\mu X^M \partial_\nu X^N \left(\mathbf{A}^{-1}\right)^{\nu\mu} \sqrt{-\det \mathbf{A}} - \\ &-\partial_\mu \left[\tau_p e^{-\Phi} \left\{ g_{KM} \partial_\nu X^M \left(\mathbf{A}^{-1}\right)_S^{\nu\mu} + b_{KM} \partial_\nu X^M \left(\mathbf{A}^{-1}\right)_A^{\nu\mu} \right\} \sqrt{-\det \mathbf{A}} \right] = 0 \,, \end{aligned} \tag{2.6}$$

while the equation of motion for the gauge field A_ν takes the form

$$\partial_\mu \left[\tau_p e^{-\Phi} (2\pi\alpha') \left(\mathbf{A}^{-1}\right)_A^{\nu\mu} \sqrt{-\det \mathbf{A}}\right] = 0 \,, \tag{2.7}$$

where the symmetric and anti-symmetric part of the $\left(\mathbf{A}^{-1}\right)^{\nu\mu}$, respectively, are given by

$$\left(\mathbf{A}^{-1}\right)_S^{\nu\mu} = \frac{1}{2} \left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} + \left(\mathbf{A}^{-1}\right)^{\mu\nu} \right) , \left(\mathbf{A}^{-1}\right)_A^{\nu\mu} = \frac{1}{2} \left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} - \left(\mathbf{A}^{-1}\right)^{\mu\nu} \right) . \tag{2.8}$$

Let us now consider the D1-brane that wraps θ_1 direction and study its dynamics when all the worldvolume modes depend on time only. More precisely, we fix the gauge as

$$\theta_1 = \xi^1 \,, \xi^0 = t = X^0 \tag{2.9}$$

and also take $A_0 = 0$. Let us also presume that $\rho = \rho(t)$. Then the matrix $\mathbf{A}$ is equal to

$$\begin{aligned} \mathbf{A}_{00} &= -L^2 \cosh^2 \rho + L^2 \dot\rho^2 \,, \\ \mathbf{A}_{01} &= -L^2 \sinh^2 \rho + (2\pi\alpha') \dot A_{\theta_1} \,, \\ \mathbf{A}_{10} &= L^2 \sinh^2 \rho - (2\pi\alpha') \dot A_{\theta_1} \,, \\ \mathbf{A}_{11} &= L^2 \sinh^2 \rho \,, \end{aligned} \tag{2.10}$$

where $\dot f = \frac{df}{dt}$. Consequently we get

$$\det \mathbf{A} = \det \tilde g + \mathcal{F}_{t\theta_1}^2 \,, \tag{2.11}$$

where

$$\det \tilde{g} = -L^4 \cosh^2 \rho \sinh^2 \rho + L^4 \sinh^2 \rho \dot{\rho}^2 \,, \mathcal{F}_{t\theta_1} = -L^2 \sinh^2 \rho + (2\pi\alpha')\dot{A}_{\theta_1} \tag{2.12}$$

and also

$$\left(\mathbf{A}^{-1}\right) = \frac{1}{\det \mathbf{A}} \begin{pmatrix} \tilde{g}_{\theta_1\theta_1} & -\mathcal{F}_{t\theta_1} \\ \mathcal{F}_{t\theta_1} & \tilde{g}_{tt} \end{pmatrix} \tag{2.13}$$

Now the equation of motion for A gives

$$\frac{(2\pi\alpha')\tau_1 \mathcal{F}_{t\theta_1}}{g_s\sqrt{-\det \tilde{g} - \mathcal{F}_{t\theta_1}^2}} = -q \Rightarrow \mathcal{F}_{t\theta_1}^2 = -\frac{g_s^2 q^2 \det \tilde{g}}{g_s^2 q^2 + (2\pi\alpha')^2 \tau_1^2} \,. \tag{2.14}$$

We must also check that the equation of motion (2.6) are obeyed for the ansatz (2.9). For $K = \theta_1$ the equation (2.6) is trivially satisfied since now all the modes do not depend on θ_1. On the other hand the equation of motion for X^0 gives

$$\partial_0 \left[\frac{\tau_1}{g_s} g_{tt} \left(\mathbf{A}^{-1}\right)_S^{00} \sqrt{-\det \mathbf{A}}\right] + \partial_0 \left[\frac{\tau_1}{g_s} b_{t\theta_1} \left(\mathbf{A}^{-1}\right)_A^{\theta_1 t} \sqrt{-\det \mathbf{A}}\right] = 0 \tag{2.15}$$

and hence we obtain the conserved quantity

$$\frac{E}{2\pi} = \frac{-g_{tt} g_{\theta_1\theta_1} \sqrt{q^2 + (2\pi\alpha')^2 \tau_1^2 g_s^{-2}} - q b_{t\theta_1} \sqrt{-\det \tilde{g}}}{(2\pi\alpha')\sqrt{-\det \tilde{g}}} \tag{2.16}$$

using (2.14). Some comments regarding the definition of the conserved quantity E is in order now. Here E means the conserved energy of the D1-brane that arises by simply integrating over θ_1 direction which implies (for homogeneous worldvolume fields) that E is proportional to 2π. We have further included the factor $e^{-\Phi} = e^{-\Phi_0} = \frac{1}{g_s}$, where g_s is the string coupling constant. This factor is important for the later comparison with the D0-brane equations of motion which we derive in the next section.

Let us try to evaluate the energy on the solution [5]

$$\cosh \rho \cos t = C \,, C > 0 \,. \tag{2.17}$$

Firstly, we have

$$-\det \tilde{g} = \frac{L^4 C^2}{\cos^4 t}(C^2 - 1) \,. \tag{2.18}$$

Then the expression for energy is

$$\frac{E}{2\pi} = L^2 \sinh^2 \rho \left(\frac{C\sqrt{q^2(2\pi\alpha')^{-2} + \tau_1^2 g_s^{-2}} + q\sqrt{C^2 - 1}}{\sqrt{C^2 - 1}}\right) \,. \tag{2.19}$$

As we have determined above E has to be conserved, but on the other hand we see that it depends explicitly on $\sinh \rho$. So the only possibility for it to be conserved is that it has to vanish. This occurs when

$$C\sqrt{q^2(2\pi\alpha')^{-2} + \tau_1^2 g_s^{-2}} + \frac{q}{(2\pi\alpha')}\sqrt{C^2 - 1} = 0 \tag{2.20}$$

and this implies

$$q^2 = -C^2(2\pi\alpha')^2 \frac{\tau_1^2}{g_s^2} . \tag{2.21}$$

So we obtain the well known result that the dS_2-brane corresponds to the unphysical situation when the electric flux on its worldvolume is purely imaginary.

Let us now return to the equation of conserved energy and try to solve it explicitly. Using the conserved energy given in (2.16) we get

$$\dot{\rho}^2 = -\frac{\cosh^4\rho \sinh^2\rho \left(\frac{L^4\tau_0^2}{g_s^2\alpha'} + \frac{q^2L^4}{\alpha'^2}\right)}{(E - \frac{L^2}{\alpha'}q\sinh^2\rho)^2} + \cosh^2\rho \tag{2.22}$$

using

$$\tau_1^2 = \frac{1}{4\pi^2\alpha'}\tau_0^2 . \tag{2.23}$$

The differential equation above can be solved explicitly, however it leads to the very complicated result, which we don't wish to present here. We will only briefly discuss the properties of given solution when we presume that E, q are real and also $\dot{\rho}^2 > 0$. Then the equation (2.22) implies following bound for ρ

$$-\frac{L^4\cosh^4\rho\sinh^2\rho(\tau_1^2 g_s^{-2} + (2\pi\alpha')^{-2}q^2)}{(\frac{E}{2\pi} - q(2\pi\alpha')^{-1}L^2\sinh^2\rho)^2} + \cosh^2\rho > 0 . \tag{2.24}$$

Solving this inequality leads to the condition

$$\sinh^2\rho \in (0, \sinh^2\rho_+) , \tag{2.25}$$

where $\sinh^2\rho_+$ is a root of the quadratic equation given above. In other words, for real E and q, we obtain motion in the finite interval and D1-brane cannot reach the boundary of AdS_3.

Instead of studying the properties of the classical trajectory of D1-brane in more detail we rather turn our attention to the possibility of explaining these unphysical solutions with imaginary electric flux in the T-dual set up.

3. T-dual background

On the other hand it was argued recently that such a configuration could be related to T-dual situation where it could correspond to Dirichlet S-brane. To make this statement more clear and precise, let us apply T-duality along θ_1 direction. More precisely, the action of T -duality along the symmetry direction θ_1 maps the string frame metric to string frame metric [20]

$$\begin{aligned}
d^2\tilde{s} &= \alpha'\left[g_{\mu\nu} - \frac{1}{g_{\theta_1\theta_1}}(g_{\mu\theta_1}g_{\theta_1\nu} - B_{\mu\theta_1}B_{\nu\theta_1})\right]dx^\mu dx^\nu + 2\frac{1}{g_{\theta_1\theta_1}}B_{\theta_1\mu}d\theta_1 dx^\nu + \frac{1}{g_{\theta_1\theta_1}}d\theta_1^2 , \\
\tilde{B} &= \frac{\alpha'}{2}dx^\mu\wedge dx^\nu\left[B_{\mu\nu} - \frac{1}{g_{\theta_1\theta_1}}(g_{\mu\theta_1}B_{\theta_1\nu} + B_{\mu\theta_1}g_{\theta_1\nu})\right] + \frac{\alpha'}{g_{\theta_1\theta_1}}g_{\theta_1\mu}d\theta_1\wedge dx^\mu , \\
\tilde{\phi} &= \phi - \frac{1}{2}\log g_{\theta_1\theta_1} .
\end{aligned} \tag{3.1}$$

where we have included in the original components of the metric $g_{\mu\nu}$ and the anti-symmetric tensor $B_{\mu\nu}$, the dimensionless factor $\frac{L^2}{\alpha'}$.

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Now we are ready to perform the T-duality along θ_1 directions. Recall that in our convention θ_1 is dimensionless and periodic with period 2π. In T-dual background we rename θ_1 as z that is still periodic with period 2π. Finally we write T-dual components of the metric g and the anti-symmetric B with the factor α'. Then the metric components of the dual background take the form (We denote the dual variable to θ_1 as z)

$$\tilde{g}_{zz} = \frac{\alpha'^2}{L^2 \sinh^2 \rho} \, , \tilde{g}_{tt} = -L^2 \, , \tilde{g}_{tz} = \tilde{g}_{zt} = \alpha' \, , \tilde{g}_{\rho\rho} = L^2 \tag{3.2}$$

while the other components of the metric remain unchanged. We also get new components of the anti-symmetric B field

$$\tilde{B}_{zt} = -L^2 \, . \tag{3.3}$$

Finally, we also obtain nonzero value of the dilaton in the form

$$\tilde{\phi} = \phi_0 - \frac{1}{2} \ln g_{\theta_1\theta_1} = \phi_0 - \ln \frac{L}{\sqrt{\alpha'}} \sinh \rho \, . \tag{3.4}$$

Under T-duality the D1-brane that wraps the circle is mapped to the D0-brane that moves around this circle. Recall that dynamics of the D0-brane is governed by the action

$$S = -\tau_0 \int d\tau e^{-\Phi} \sqrt{-\mathbf{A}} \, , \mathbf{A} = g_{MN} \dot{X}^M \dot{X}^N \, , \tag{3.5}$$

where in the following we omit the tilde on g. The equations of motion for X^M that follow from the action (3.5) take the form

$$\partial_K \left[e^{-\Phi} \right] \sqrt{-\mathbf{A}} - \frac{1}{2} e^{-\Phi} \partial_K g_{MN} \dot{X}^M \dot{X}^N \frac{1}{\sqrt{-\mathbf{A}}} + \frac{d}{d\tau} \left[e^{-\Phi} \frac{g_{KM} \dot{X}^M}{\sqrt{-\mathbf{A}}} \right] = 0 \, . \tag{3.6}$$

Now we fix the gauge that the worldvolume parameter τ is equal to $t \equiv X^0$. Then $\mathbf{A}$ is equal to

$$\mathbf{A} = g_{tt} + g_{\rho\rho} \dot{\rho}^2 + 2 g_{tz} \dot{Z} + g_{zz} \dot{Z}^2 \tag{3.7}$$

and also the equation of motion for $X^0 = \tau$ takes the form

$$\frac{d}{d\tau} \left[\frac{e^{-\Phi} (g_{00} + g_{tz} \dot{Z})}{\sqrt{-\mathbf{A}}} \right] = 0 \tag{3.8}$$

that implies that the quantity in the bracket is conserved. As usual it is useful to make use of the Hamiltonian formalism after fixing the gauge. To do this we observe that the Lagrangian has the form

$$\mathcal{L} = -\sqrt{V - \sum_i (f_i (\partial_0 \Phi^i)^2 + B_i \partial_0 \Phi^i)} \equiv -\triangle \, , \tag{3.9}$$

where V contain scalar potential for various fields Φ^i. The conjugate momentum P_i to Φ_i takes the form

$$P_i = \frac{\delta \mathcal{L}}{\delta \partial_0 \Phi^i} = \frac{2 f_i \partial_0 \Phi^i + B_i}{2\triangle} \, , \qquad \partial_0 \Phi^i = \frac{1}{2 f_i} (2 P_i \triangle - B_i) \tag{3.10}$$

so that the Hamiltonian is equal to

$$H = \sum_i P_i \partial_0 \Phi^i - \mathcal{L} = \frac{2V + \sum \frac{B_i^2}{2f_i}}{2\triangle} - \sum_i \frac{B_i}{2f_i} P_i$$
$$= \sqrt{\left(V + \sum_i \frac{B_i^2}{4f_i}\right)\left(1 + \sum_i \frac{P_i^2}{f_i}\right)} - \sum_i \frac{B_i P_i}{2f_i}, \tag{3.11}$$

where on the second line we have expressed the Hamiltonian as a function of canonical variables Φ^i, P_i. Returning to the action (3.5) we obtain

$$V = -e^{-2\Phi}\tau_0^2 g_{tt}\,,\, f_z = e^{-2\Phi}\tau_0^2 g_{zz}\,,\, f_\rho = e^{-2\Phi}\tau_0^2 g_{\rho\rho}\,,\, B_z = 2e^{-2\Phi}\tau_0^2 g_{zt} \tag{3.12}$$

and hence the Hamiltonian is equal to

$$H = \frac{1}{\sqrt{g_{zz}}}\sqrt{(-g_{tt}g_{zz} + g_{tz}^2)\left(e^{-2\Phi}\tau_0^2 + \frac{1}{g_{zz}}P_z^2 + \frac{1}{g_{\rho\rho}}P_\rho^2\right)} - \frac{g_{zt}}{g_{zz}}P_z\,. \tag{3.13}$$

Firstly, since the Hamiltonian does not explicitly depend on Z it implies that P_z is constant of motion

$$\dot{P}_z = -\frac{\delta H}{\delta Z} = 0\,. \tag{3.14}$$

On the other hand the equation of motion for ρ is

$$\dot{\rho} = \frac{\delta H}{\delta P_\rho} = \frac{1}{g_{zz}g_{\rho\rho}}(-g_{tt}g_{zz} + g_{tz}^2)\frac{P_\rho}{E + \frac{g_{zt}}{g_{zz}}P_z}\,. \tag{3.15}$$

As usual we simplify this equation using the fact that the Hamiltonian is conserved and equal to energy E. Then we express from (3.13) P_ρ as

$$P_\rho^2 = \frac{1}{1+\sinh^2\rho}\left(E + \frac{L^2}{\alpha'}P_z\sinh^2\rho\right)^2 - \left(\frac{L^4\tau_0^2}{\alpha' g_s^2}\sinh^2\rho + \frac{L^4}{\alpha'^2}P_z^2\sinh^2\rho\right) \tag{3.16}$$

using the explicit metric components given above and also the fact that $e^{-2\Phi} = \frac{1}{g_s^2}\sinh^2\rho$. Then we obtain

$$\dot{\rho}^2 = -\frac{\cosh^4\rho\sinh^2\rho}{(E + \frac{L^2}{\alpha'}P_z\sinh^2\rho)^2}\left(\frac{L^4\tau_0^2}{\alpha' g_s^2} + \frac{L^4}{\alpha'^2}P_z^2\right) + \cosh^2\rho\,. \tag{3.17}$$

Now the equation (3.17) describes the dynamics of D0-brane in dual background. As we expect this equation is the same as the equation that determines the dynamics of D1-brane in the original background. In fact, we see that this has the same form as the equation (2.22) if we identify

$$P_z^2 = q^2\,. \tag{3.18}$$

Naively we can say that this is the correct quantization condition for the motion of a test D0-brane along a compact direction of periodicity 2π. Of course there is an important issue that the momentum is imaginary and hence the wave function of D0-brane is not periodic in z variable[3]. We should rather claim that the momentum P_z is conserved with the value given in (3.18).

We can also see that the energy is the same in both cases. Then it immediately follows that the classical trajectory $\cosh\rho\cos t = C$ which corresponds to imaginary P_z is again unphysical while E is equal to zero. Note also that the equation of motion for z takes the form

$$\dot{Z} = \frac{\delta H}{\delta P_z} = \frac{1}{g_{zz}^2}(-g_{tt}g_{zz} + g_{tz}^2)\frac{P_z}{E + \frac{g_{zt}}{g_{zz}}P_z} - \frac{g_{zt}}{g_{zz}} \tag{3.19}$$

that for $E = 0\,, P_z = -q$, reduces to

$$\dot{Z} = -\frac{g_{tt}}{g_{zz}} = \frac{L^2}{\alpha'}\,. \tag{3.20}$$

We see that the velocity v_z is constant and it does not depend on the value of the charge q. This result is a consequence of the fact that the energy E is zero for the trajectory $\cosh\rho\cos t = C$ as can be seen easily from the form of the equation (3.19). On the other hand it is also clear that when $E \neq 0$ the motion along z direction will depend on P_z.

In summary, in the T-dual picture in case of ordinary D0-brane we once again obtain a situation which is unphysical. Then, following [4] we can expect that in T-dual background the object, that is obtained in the dual picture of the corresponding $D1$-brane will be a $S0$-brane. To see this explicitly we perform in the next section the analysis of the time-dependent tachyon condensation on unstable Dp-brane in general background. We argue that there exists a singular time dependent tachyon solution which leads to the emergence of $S(p-1)$-brane that has imaginary charge with respect to NS-NS fields however has real charge with respect to the Ramond-Ramond (RR) fields.

4. S(p-1)-brane in general background

This section is devoted to the study of the singular time dependent tachyon condensation on the world volume of non-BPS Dp-brane that leads to the emergence of S(p-1)-brane.

Once again, we begin with the Dirac-Born-Infeld like tachyon effective action in general background [21–24] [4]

$$\begin{aligned}
&S = -\int d^{p+1}\xi e^{-\Phi}V(T)\sqrt{-\det \mathbf{A}}\,, \\
&\mathbf{A}_{\mu\nu} = g_{MN}\partial_\mu X^M\partial_\nu X^N + b_{MN}\partial_\mu X^M\partial_\nu X^N + F_{\mu\nu} + \partial_\mu T\partial_\nu T\,, \mu\,,\nu = 0,\ldots,p\,, \\
&F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,
\end{aligned} \tag{4.1}$$

[3]In any case, if one computes the squared norm of the tangent vector to the D0-brane trajectory, this gives $\left(-\frac{L^2\cosh^4\rho}{P_z^2 g_s^2\sinh^2\rho}\right)$. Therefore if P_z is imaginary this corresponds to supernuminal signature.

[4]We will work in this section in units $(2\pi\alpha') = 1$.

where $A_\mu\,,\mu,\nu=0,\dots,p$ and $X^{M,N}\,,M,N=0,\dots,9$ are gauge and the transverse scalar fields on the worldvolume of the non-BPS Dp-brane and T is the tachyon field. $V(T)$ is the tachyon potential that is symmetric under $T\to -T$ has maximum at $T=0$ equal to the tension of a non-BPS Dp-brane τ_p and has its minimum at $T=\pm\infty$ where it vanishes.

We must also stress that there exists a Wess-Zumino term for non-BPS Dp-brane that expresses the coupling of this Dp-brane to the Ramond-Ramond fields. [25 − 29] that is expected to have the form

$$S_{WZ}=\int_\Sigma V(T)dT\wedge Ce^{F+B}\,, \tag{4.2}$$

where Σ denotes the worldvolume of non-BPS Dp-brane and C collects all RR n-form gauge potentials (pulled back to brane worldvolume).

In what follows we closely follow the analysis performed in [30]. As usual we start to solve the equations of motion for T,X^M and A_μ. The equation of motion for tachyon takes the form

$$-e^{-\Phi}V'(T)\sqrt{-\det\mathbf{A}}+\partial_\mu\left[e^{-\Phi}\partial_\nu T\left(\mathbf{A}^{-1}\right)^{\nu\mu}_S\sqrt{-\det\mathbf{A}}\right]+J_T=0\,, \tag{4.3}$$

where $J_T=\frac{\delta}{\delta T}S_{WZ}$. For scalar modes we obtain

$$\begin{aligned}&-\frac{\delta e^{-\Phi}}{\delta X^K}V\sqrt{-\det\mathbf{A}}-\\&-\frac{e^{-\Phi}}{2}V\left(\frac{\delta g_{MN}}{\delta X^K}\partial_\mu X^M\partial_\nu X^N+\frac{\delta b_{MN}}{\delta X^K}\partial_\mu X^M\partial_\nu X^N\right)\left(\mathbf{A}^{-1}\right)^{\nu\mu}\sqrt{-\det\mathbf{A}}+\\&+\partial_\mu\left[e^{-\Phi}Vg_{KM}\partial_\nu X^M\left(\mathbf{A}^{-1}\right)^{\nu\mu}_S\sqrt{-\det\mathbf{A}}\right]+\\&\partial_\mu\left[e^{-\Phi}Vb_{KM}\partial_\nu X^M\left(\mathbf{A}^{-1}\right)^{\nu\mu}_A\sqrt{-\det\mathbf{A}}\right]+J_K=0\,,\end{aligned} \tag{4.4}$$

where $J_K=\frac{\delta}{\delta X^K}S_{WZ}$. Finally, the equations of motion for A_μ are given by

$$\partial_\nu\left[e^{-\Phi}V\left(\mathbf{A}^{-1}\right)^{\mu\nu}_A\sqrt{-\det\mathbf{A}}\right]+J^\mu=0\,, \tag{4.5}$$

where $J^\mu=\frac{\delta}{\delta A_\mu}S_{WZ}$. Now we try to find the solution of the equations of motion (4.3), (4.4) and (4.5) that can be interpreted as a lower dimensional S(p-1)-brane. More precisely, we can show that the dynamics of the kink is governed by the equations of motion that arise from the action for S(p-1)-brane in general background

$$\begin{aligned}S&=S^S_{DBI}+S^S_{WZ}\,,\\S^S_{DBI}&=-T_{S(p-1)}\int d^p\xi e^{-\Phi}\sqrt{\det\mathbf{a}}\,,\\S^S_{WZ}&=\mu_{S(p-1)}\sum_{n\geq 0}\frac{1}{n!(2!)^n(2p-2n)!}\int d^p\xi\epsilon^{\alpha_1\dots\alpha_p}(\tilde{\mathcal{F}})^n_{\alpha_1\dots\alpha_{2n}}\tilde{\mathcal{C}}_{\alpha_{2n+1}\dots\alpha_p}\,,\end{aligned} \tag{4.6}$$

where

$$
\begin{aligned}
\mathbf{a}_{\alpha\beta} &= (g_{MN}+b_{MN})\partial_\alpha X^M \partial_\beta X^N + F_{\alpha\beta}\,, \\
\tilde{\mathcal{F}}_{\alpha\beta} &= F_{\alpha\beta} + b_{MN}\partial_\alpha X^M \partial_\beta X^N\,, \\
\tilde{\mathcal{C}}_{\alpha_{2n+1}\dots\alpha_p} &= C_{M_{2n+1}\dots M_p}\partial_{\alpha_{2n+1}} X^{M_{2n+1}}\dots\partial_{\alpha_p}X^{M_p}
\end{aligned} \tag{4.7}
$$

and $\xi^\alpha, \alpha = 1,\dots,p$. Finally, $T_{S(p-1)}$ is S(p-1)-brane tension and $\mu_{S(p-1)}$ is the charge of S(p-1)-brane with respect to Ramond-Ramond fields. These quantities will be determined during the calculations.

In other words we will show that the modes given in (4.14) that propagate on the worldvolume of the kink obey the equations of motion derived from (4.6) that have the form

$$
\begin{aligned}
&- T_{S(p-1)}\frac{\delta e^{-\Phi}}{\delta X^K}\sqrt{\det\mathbf{a}} - \\
&-T_{S(p-1)}\frac{e^{-\Phi}}{2}\left(\frac{\delta g_{MN}}{\delta X^K}\partial_\alpha X^M\partial_\beta X^N + \frac{\delta b_{MN}}{\delta X^K}\partial_\alpha X^M \partial_\beta X^N\right)(\mathbf{a}^{-1})^{\beta\alpha}\sqrt{\det\mathbf{a}} + \\
&+T_{S(p-1)}\partial_\alpha\left[e^{-\Phi}g_{KM}\partial_\beta X^M(\mathbf{a}^{-1})^{\beta\alpha}_S\sqrt{\det\mathbf{a}}\right] + \\
&+T_{S(p-1)}\partial_\alpha\left[e^{-\Phi}b_{KM}\partial_\beta X^M(\mathbf{a}^{-1})^{\beta\alpha}_A\sqrt{\det\mathbf{a}}\right] + \tilde{J}_K = 0\,,
\end{aligned} \tag{4.8}
$$

where

$$
\begin{aligned}
\tilde{J}_K &= \frac{\delta S_{WZ}}{\delta X^K} \\
&= \mu_{S(p-1)}\sum_{n\geq 0}\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\alpha_1\dots\alpha_p}\Big[\partial_K b_{MN}\partial_{\alpha_1}X^M\partial_{\alpha_2}X^N(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\dots\alpha_{2n}}\tilde{\mathcal{C}}_{\alpha_{2n+1}\dots\alpha_p} \\
&+(\tilde{\mathcal{F}})^n_{\alpha_1\dots\alpha_{2n}}\partial_K\tilde{\mathcal{C}}_{M_1\dots M_{2p-2n}}\partial_{\alpha_{2n+1}}X^{M_1}\dots\partial_{\alpha_p}X^{M_{2p-2n}} - \\
&-2n\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}X^M(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\dots\alpha_{2n}}\tilde{\mathcal{C}}_{\alpha_{2n+1}\dots\alpha_p}\right] - \\
&-(2p-2n)\partial_{\alpha_{2n+1}}\left[(\tilde{\mathcal{F}})^n_{\alpha_1\dots\alpha_{2n}}C_{KM_2\dots M_{2p-2n}}\partial_{\alpha_{2n+2}}X^{M_2}\dots\partial_{\alpha_p}X^{M_{2p-2n}}\right]\Big]\,.
\end{aligned} \tag{4.9}
$$

In the same way we get that the equation of motion for A_α are

$$
T_{S(p-1)}\partial_\beta\left[e^{-\Phi}(\mathbf{a}^{-1})^{\alpha\beta}_A\sqrt{-\det\mathbf{a}}\right] + \tilde{J}^\alpha = 0\,, \tag{4.10}
$$

where

$$
\tilde{J}^{\alpha_1} = \mu_{S(p-1)}\sum_{n\geq 0}\frac{2n}{n!2^n(2p-2n)!}\epsilon^{\alpha_1\dots\alpha_p}\partial_{\alpha_2}\left[(\tilde{\mathcal{F}})^{n-1}_{\alpha_3\dots\alpha_{2n}}\tilde{\mathcal{C}}_{\alpha_{2n+2}\dots\alpha_p}\right]\,. \tag{4.11}
$$

In what follows we will proceed in the same way as in [30] so we can be brief and recommend the paper [30] for more details.

We begin with the presumption that the tachyon kink depends on the time coordinate on the worldvolume of Dp-brane. We will also see that when we consider the singular limit we obtain the formal solution that leads to the negative expression under square root. In spite this fact we will argue that this singular solution describes S(p-1)-brane.

More precisely, let us consider the following ansatz for tachyon [31]

$$T(x,\xi) = f(a(x-t(\xi)))\,, \tag{4.12}$$

where x is time coordinate on the worldvolume of Dp-brane and where t is some unknown function of the $\xi^\alpha\,, \alpha = 1,\dots,p$ euclidean coordinates on the kink. We also presume as in [31] that $f(u)$ satisfies following properties

$$f(-u) = -f(u)\,, f'(u) > 0\,, \forall u\,, f(\pm\infty) = \pm\infty \tag{4.13}$$

but is otherwise an arbitrary function of its argument u. a is a constant that we shall take to ∞ in the end. In this limit we have $T = \infty$ for $x > t(\xi)$ and $T = -\infty$ for $x < t(\xi)$. Let us also presume following ansatz for the massless fields

$$X^M(x,\xi) = X^M(\xi)\,, A_x(x,\xi) = 0\,, A_\alpha(x,\xi) = A_\alpha(\xi)\,, \alpha = 1,\dots,p\,. \tag{4.14}$$

With this ansatz the matrix $\mathbf{A}_{\mu\nu}$ takes the form

$$\mathbf{A} = \begin{pmatrix} a^2 f'^2 & -a^2 f'^2 \partial_\beta t \\ -a^2 f'^2 \partial_\alpha t & \mathbf{a}_{\alpha\beta} + a^2 f'^2 \partial_\alpha t \partial_\beta t \end{pmatrix}\,, \tag{4.15}$$

where

$$\mathbf{a}_{\alpha\beta} = (g_{MN} + b_{MN})\partial_\alpha X^M \partial_\beta X^N + F_{\alpha\beta}\,. \tag{4.16}$$

Now using the fact that

$$\det \mathbf{A} = \det\left(\mathbf{A}_{\alpha\beta} - \mathbf{A}_{\alpha x}\frac{1}{\mathbf{A}_{xx}}\mathbf{A}_{x\beta}\right)\det \mathbf{A}_{xx} \tag{4.17}$$

we get

$$\det \mathbf{A} = a^2 f'^2 \det \mathbf{a}\,. \tag{4.18}$$

As a next step we determine the inverse matrix $\left(\mathbf{A}^{-1}\right)$. After some calculations we get the result

$$\begin{aligned} \left(\mathbf{A}^{-1}\right)^{\alpha\beta} &= (\mathbf{a}^{-1})^{\alpha\beta} \ \left(\mathbf{A}^{-1}\right)^{x\beta} = \partial_\alpha t (\mathbf{a}^{-1})^{\alpha\beta}\,, \\ \left(\mathbf{A}^{-1}\right)^{\alpha x} &= (\mathbf{a}^{-1})^{\alpha\beta}\partial_\beta t\,, \left(\mathbf{A}^{-1}\right)^{xx} = \partial_\alpha t (\mathbf{a}^{-1})^{\alpha\beta}\partial_\beta t \end{aligned} \tag{4.19}$$

For next purposes following relation will be also useful

$$\left(\mathbf{A}^{-1}\right)^{\mu x}_S - \left(\mathbf{A}^{-1}\right)^{\mu\alpha}_S \partial_\alpha t = \frac{1}{a^2 f'^2}\left(\delta^\mu_x - \left(\mathbf{A}^{-1}\right)^{x\mu}_S\right)\,. \tag{4.20}$$

With the help of this expression we get

$$\partial_\mu\left[e^{-\Phi} V \partial_\nu T \left(\mathbf{A}^{-1}\right)^{\nu\mu}_S \sqrt{-\det \mathbf{A}}\right] = V' a f' e^{-\Phi}\sqrt{-\det \mathbf{a}} - V\partial_\alpha\left[e^{-\Phi}(\mathbf{a}^{-1})^{\beta\alpha}_S \partial_\beta t \sqrt{-\det \mathbf{a}}\right]\,, \tag{4.21}$$

where we have used the fact that the only field that depends on x is a tachyon. Then the DBI part [5] of the tachyon equation of motion (4.3) takes the form

$$\partial_\alpha \left[V(f)e^{-\Phi}(\mathbf{a}^{-1})^{\beta\alpha}_S\partial_\beta t\sqrt{-\det\mathbf{a}}\right] = i\partial_\alpha \left[V(f)e^{-\Phi}(\mathbf{a}^{-1})^{\beta\alpha}_S\partial_\beta t\sqrt{\det\mathbf{a}}\right] . \tag{4.22}$$

Now we consider the DBI part of the equation of motion for X^K (4.4). In the same way as in [30] we can show the the DBI part of the equation of motion (4.4) takes the form

$$\begin{aligned}&\sqrt{-1}af'V\left(-\partial_K[e^{-\Phi}]\sqrt{\det\mathbf{a}} - \frac{e^{-\Phi}}{2}\left(g_{MN,K}+b_{MN,K}\right)\partial_\alpha X^M\partial_\beta X^N(\mathbf{a}^{-1})^{\alpha\beta}\sqrt{\det\mathbf{a}}\right.\\ &\left.+\partial_\beta\left[e^{-\Phi}g_{KM}\partial_\alpha X^M(\mathbf{a}^{-1})^{\alpha\beta}_S\sqrt{\det\mathbf{a}}\right] + \partial_\beta\left[e^{-\Phi}b_{KM}\partial_\alpha X^M(\mathbf{a}^{-1})^{\beta\alpha}_A\sqrt{\det\mathbf{a}}\right]\right) .\end{aligned} \tag{4.23}$$

Now let us consider the DBI part of the equation of motion for gauge field (4.5). For A_x we get

$$\partial_\nu\left[Ve^{-\Phi}\left(\mathbf{A}^{-1}\right)^{x\nu}_A\sqrt{-\det\mathbf{A}}\right] = \sqrt{-1}af'V\partial_\alpha\left[e^{-\Phi}(\mathbf{a}^{-1})^{\beta\alpha}_A\partial_\beta t\sqrt{\det\mathbf{a}}\right]. \tag{4.24}$$

On the other hand the equations of motion for A_α take the form

$$\partial_\mu\left[e^{-\Phi}\left(\mathbf{A}^{-1}\right)^{\alpha\mu}_A\sqrt{-\det\left(\mathbf{A}^{-1}\right)}\right] = \sqrt{-1}af'V\partial_\beta\left[e^{-\Phi}(\mathbf{a}^{-1})^{\alpha\beta}_A\sqrt{\det\mathbf{a}}\right]. \tag{4.25}$$

As a next step we evaluate the currents J_T, J_K and J^{μ_1}. for the ansatz (4.12) and (4.14). It was shown in [30] following [32] that these currents take the form

$$J^{\mu_1} = \sum_{n\geq 0}\frac{2n}{n!2^n(2p-2n)!}\epsilon^{\mu_1\dots\mu_{p+1}}\partial_{\mu_2}\left[V(T)(\mathcal{F})^{n-1}_{\mu_3\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\partial_{\mu_{p+1}}T\right], \tag{4.26}$$

$$\begin{aligned}J_T =& \sum_{n\leq 0}\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\mu_1\dots\mu_{p+1}}V'(T)\left((\mathcal{F})^n_{\mu_1\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\partial_{\mu_{p+1}}T\right) -\\ &-\partial_{\mu_{p+1}}\sum_{n\leq 0}\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\mu_1\dots\mu_{p+1}}\left[V(T)(\mathcal{F})^n_{\mu_1\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\right]\end{aligned} \tag{4.27}$$

and

$$\begin{aligned}J_K =& \sum_{n\leq 0}\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\mu_1\dots\mu_{p+1}}\\ &\times[V(T)b_{MN,K}\partial_{\mu_1}X^M\partial_{\mu_2}X^N(\mathcal{F})^{n-1}_{\mu_3\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\partial_{\mu_{p+1}}T\\ &+V(T)(\mathcal{F})^n_{\mu_1\dots\mu_{2n}}\partial_K C_{M_1\dots M_{2p-2n}}\partial_{\mu_{2n+1}}X^{M_1}\dots\partial_{\mu_p}X^{M_{2p-2n}}\partial_{\mu_{p+1}}T -\\ &-2\partial_{\mu_1}[V(T)b_{KM}\partial_{\mu_2}X^M(\mathcal{F})^{n-1}_{\mu_3\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\partial_{\mu_{p+1}}T] -\\ &-(2p-2n)\partial_{2n+1}[V(T)(\mathcal{F})^n_{\mu_1\dots\mu_{2n}}C_{KM_2\dots M_{2p-2n}}\partial_{\mu_{2n+2}}X^{M_2}\dots\partial_{\mu_p}X^{M_{2p-2n}}\partial_{\mu_{p+1}}T]] .\end{aligned} \tag{4.28}$$

Let us start with J_T that can be written as

$$J_T = -\sum_{n\leq 0}V(T)\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\mu_1\dots\mu_{p+1}}\partial_{\mu_{p+1}}\left((\mathcal{F})^n_{\mu_1\dots\mu_{2n}}C_{\mu_{2n+1}\dots\mu_p}\right). \tag{4.29}$$

[5] "DBI" part of the equation of motion means the part that arises from the variation of the DBI action.

As the first step we determine the components of the embedding of various fields. It can be shown [30] that the only nonzero components of $\mathcal{F}_{\mu\nu}$ are $\mathcal{F}_{\alpha\beta}$. For $C^{(n)}$ the situation is the same, namely any component with x index is equal to zero. Then it can be easily shown that the tachyon current is equal to zero [30].

Now we consider the gauge current J^μ. Firstly, it can be easily shown [30] that J^x is equal to

$$J^x = -af'V \sum_{n\geq 0} \frac{2n}{n!2^n(2p-2n)!} \epsilon^{x\alpha_1\ldots\alpha_p} \partial_{\alpha_1} \left[(\mathcal{F})^{n-1}_{\alpha_3\ldots\alpha_{2n}} C_{\alpha_{2n+1}\ldots\alpha_{p-1}} \partial_{\alpha_p} t \right] \tag{4.30}$$

while J^{α_1} takes the form

$$J^{\alpha_1} = \sum_{n\geq 0} af'V \frac{2n}{n!2^n(2p-2n)!} \epsilon^{\alpha_1\alpha_2\ldots\alpha_p x} \partial_{\alpha_2} \left[(\mathcal{F})^{n-1}_{\mu_3\ldots\mu_{2n}} C_{\mu_{2n+1}\ldots\mu_p} \right] = af'V\tilde{J}^{\alpha_1} \,, \tag{4.31}$$

where we have introduced the notation $\tilde{J}^{\alpha_1}$ that is a correct form of the gauge current for S(p-1)-brane. If we now combine (4.25) with (4.31) we get

$$af'V \left[i\partial_\beta \left[e^{-\Phi} (\mathbf{a}^{-1})^{\alpha\beta}_A \sqrt{\det \mathbf{a}} \right] + \tilde{J}^\alpha \right] = 0 \,. \tag{4.32}$$

Let us now analyze the behavior of the term $af'V$ in the limit $a \to \infty$. Since by definition $f'(u)$ is finite for all u it remains to study the properties of the expression aV. Since $V \sim e^{-T}$ for $T \to \infty$ we have

$$\begin{aligned} \lim_{a\to\infty} aV(f(a(x-t(\xi))) &= \text{(for } x \neq t(\xi)) \\ \lim_{a\to\infty} \frac{a}{e^{f(a(x-t(\xi)))}} &= \frac{1}{(x-t(\xi))f'} \lim_{a\to\infty} e^{-f(a(x-t(\xi)))} = 0 \,. \end{aligned} \tag{4.33}$$

We see that for $x \neq t(\xi)$ the expression aV goes to zero in the limit $a \to \infty$. On the other hand for $x = t(\xi)$ the potential $V(0) = \tau_p$ and hence in order to obey the equation of motion for A_α we find that the expression in the bracket in (4.32) should vanish. In fact, this expression is correct form of the equation of motion for A_α that propagate on the worldvolume of S(p-1)-brane.

From (4.32) we can also deduce that the S(p-1)-brane tension and its charge with respect to Ramond-Ramond fields are equal to

$$T_{S(p-1)} = iT_{p-1} \,, \mu_{S(p-1)} = \mu_{p-1} \,, \tag{4.34}$$

where T_{p-1} is tension of BPS D(p-1)-brane and μ_{p-1} is its corresponding charge. Even if the form of the equation (4.32) suggests that the tension of S(p-1)-brane can be arbitrary we will give arguments for the validity of (4.34) in the next subsection.

Let us now turn to the equation of motion for A_x. It was shown in [30] that it has solution in case when we demand that

$$\partial_\alpha t = 0 \,. \tag{4.35}$$

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The expression above also implies that the equation of motion for tachyon that for $J_T = 0$ has the form

$$V\partial_\alpha \left[\sqrt{-1}e^{-\Phi}(\mathbf{a}^{-1})_S^{\beta\alpha}\partial_\beta t\sqrt{\det \mathbf{a}}\right] = 0 \tag{4.36}$$

is obeyed.

Finally, we will consider the current J^K. Again, as was shown in [30] that for the ansatz (4.12) and (4.14) this current takes the form

$$\begin{aligned}
J_K &= af'V\sum_{n\leq 0}\frac{1}{n!(2!)^n(2p-2n)!}\epsilon^{\alpha_1\dots\alpha_p x}\left(b_{MN,K}\partial_{\alpha_1}X^M\partial_{\alpha_2}X^N(\mathcal{F})^{n-1}_{\alpha_3\dots\alpha_{2n}}C_{\alpha_{2n+1}\dots\alpha_p}\right. \\
&\quad +(\mathcal{F})^n_{\alpha_1\dots\alpha_{2n}}\partial_K C_{M_1\dots M_{2p-2n}}\partial_{\alpha_{2n+1}}X^{M_1}\dots\partial_{\alpha_p}X^{M_{2p-2n}}- \\
&\quad -2\partial_{\alpha_1}\left[b_{KM}\partial_{\alpha_2}X^M(\mathcal{F})^{n-1}_{\alpha_3\dots\alpha_{2n}}C_{\alpha_{2n+1}\dots\alpha_p}\right]+ \\
&\quad \left.+(2p-2n)\partial_{\alpha_{2n+1}}\left[(\mathcal{F})^n_{\alpha_1\dots\alpha_{2n}}C_{KM_2\dots M_{2p-2n}}\partial_{\alpha_{2n+2}}X^{M_2}\dots\partial_{\alpha_p}X^{M_{2p-2n}}\right]\right) \\
&\equiv af'V\tilde{J}^K\,.
\end{aligned} \tag{4.37}$$

Using (4.23) and (4.37) we obtain the final form of the equation of motion for X^K in the form

$$\begin{aligned}
&af'V\left(-i\partial_K[e^{-\Phi}]\sqrt{\det\mathbf{a}} - i\frac{e^{-\Phi}}{2}\left(g_{MN,K}+b_{MN,K}\right)\partial_\alpha X^M\partial_\beta X^N(\mathbf{a}^{-1})^{\alpha\beta}\sqrt{\det\mathbf{a}}\right. \\
&\left.+i\partial_\beta\left[e^{-\Phi}\left\{g_{KM}\partial_\alpha X^M(\mathbf{a}^{-1})^{\alpha\beta}_S + b_{KM}\partial_\alpha X^M(\mathbf{a}^{-1})^{\beta\alpha}_A\right\}\sqrt{\det\mathbf{a}}\right]+\tilde{J}^K\right) = 0\,.
\end{aligned} \tag{4.38}$$

Following now a discussion given below (4.32) we see that the expression in the bracket in (4.38) should be equal to zero. On the other hand this equation is exactly the equation of motion for the embedding mode that lives on the worldvolume of S(p-1)-brane.

Let us briefly discuss the meaning of the condition $\partial_\alpha t = 0$. Following [30] we can argue that all tachyon kink solutions are parameterized with the constant t that determines the core of the kink and that all t are equivalent. This is natural result since we have not fixed the gauge on the worldvolume of non-BPS Dp-brane.

4.1 Stress energy tensor

Further support for an interpretation of the tachyon kink as a lower dimensional S(p-1)-brane can be derived from the analysis of the stress energy tensor for the non-BPS Dp-brane. In order to find its form recall that we can write the action (4.1) as

$$S_p = -\int d^{10}x d^{(p+1)}\xi\delta(X^M(\xi)-x^M)e^{-\Phi}V(T)\sqrt{-\det\mathbf{A}}\,. \tag{4.39}$$

From (4.39) we can easily determine components of the stress energy tensor $T_{MN}(x)$ of an unstable D-brane using the fact that the stress energy tensor $T_{MN}(x)$ is defined as the variation of S_p with respect to $g_{MN}(x)$

$$\begin{aligned}
T_{MN}(x) &= -2\frac{\delta S_p}{\sqrt{-g(x)}\delta g^{MN}(x)} \\
&= -\int d^{(p+1)}\xi\frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}}e^{-\Phi}Vg_{MK}g_{NL}\partial_\mu X^K\partial_\nu X^L(\mathbf{A}^{-1})^{\nu\mu}_S\sqrt{-\det\mathbf{A}}\,.
\end{aligned} \tag{4.40}$$

Now from (4.12) and (4.14) we know that all massless modes are x independent. Hence (4.40) is equal to

$$\begin{aligned} T_{MN}(x) &= -\int dxaf'V(f(x))\int d^p\xi\frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}}\times \\ &\times e^{-\Phi}g_{MK}g_{NL}\partial_\alpha X^K\partial_\beta X^L(\mathbf{a}^{-1})^{\beta\alpha}_S\sqrt{-\det\mathbf{a}} = \\ &-iT_{p-1}\int d^p\xi\frac{\delta(X^M(\xi)-x^M)}{\sqrt{-g(x)}}e^{-\Phi}g_{MK}g_{NL}\partial_\alpha X^K\partial_\beta X^L(\mathbf{a}^{-1})^{\beta\alpha}_S\sqrt{\det\mathbf{a}}\,, \end{aligned} \tag{4.41}$$

where

$$T_{p-1} = \int dxaV(f)f' = \int dmV(m) \tag{4.42}$$

is a tension of BPS D(p-1)-brane. In other words the stress energy tensor evaluated on the ansatz (4.12) and (4.14) corresponds to the stress energy tensor for S(p-1)-brane. We also see that it is natural to define the tension of $S(p-1)$-brane as $T_{S(p-1)} = iT_{p-1}$.

In the same way we can calculate the charge to NS-NS two form fields. Again, since this charge follows from the variation of the DBI part of the non-BPS Dp-brane effective action we again get that this charge is imaginary. In conclusion, the effective field theory analysis of the time dependent tachyon kink suggests that S(p-1)-brane has imaginary charge with respect to NS-NS fields while is real to Ramond-Ramond fields.

5. S0-brane in the T-dual background

Now we would like to apply the general discussion given in previous section for S0-brane in the dual background defined in section (3). Recall that S0-brane action has the form

$$S = -\tau_{S0}\int d\xi e^{-\Phi}\sqrt{g_{MN}\dot{X}^M\dot{X}^N}\,, \tag{5.1}$$

where ξ is world line coordinate and X^M are embedding coordinates of S0-brane. If we vary the action (5.1) we obtain the equations of motion for X^K in the form

$$\partial_K[e^{-\Phi}]\sqrt{g_{MN}\dot{X}^M\dot{X}^N} + \frac{e^{-\Phi}\partial_K g_{MN}\dot{X}^M\dot{X}^N}{2\sqrt{g_{MN}\dot{X}^M\dot{X}^N}} - \frac{d}{d\xi}\left[\frac{e^{-\Phi}g_{MN}\dot{X}^N}{\sqrt{g_{MN}\dot{X}^M\dot{X}^N}}\right] = 0\,. \tag{5.2}$$

Since we presume that S0-brane wraps the z direction we choose the gauge

$$\xi = Z \tag{5.3}$$

and hence

$$g_{MN}\dot{X}^M\dot{X}^N = g_{zz} + 2g_{tz}\dot{T} + g_{tt}\dot{T}^2 + g_{\rho\rho}\dot{\rho}^2\,, \tag{5.4}$$

where $X^0 \equiv T\,, \dot{(\ldots)} = \frac{d(\ldots)}{d\xi}$. For convenience we write again the background fields in T-dual spacetime

$$\begin{aligned} g_{zz} &= \frac{\alpha'^2}{L^2\sinh^2\rho}\,, \qquad g_{tt} = -L^2\,, \qquad g_{tz} = g_{zt} = \alpha'\,, \qquad g_{\rho\rho} = L^2\,, \\ b_{zt} &= -L^2\,, \qquad e^{-\Phi} = \frac{L}{g_s\sqrt{\alpha'}}\sinh\rho\,. \end{aligned} \tag{5.5}$$

Then the action (5.1) takes the form

$$S = -\tau_{S0} \int dz e^{-\Phi} \sqrt{g_{zz} + 2g_{tz}\dot{T} + g_{tt}\dot{T}^2 + g_{\rho\rho}\dot{\rho}^2}\,. \tag{5.6}$$

Let us now try to find the 'dynamics' of S0-brane in T-dual background. Note that the Lagrangian has the form

$$\mathcal{L} = -\sqrt{V + \sum_i (f_i(\partial_0\Phi^i)^2 + B_i\partial_0\Phi^i)} \equiv -\triangle\,, \tag{5.7}$$

where V contain scalar potential for various fields Φ^i. Then in the same way as in the section (3) we determine the corresponding Hamiltonian

$$H = \sqrt{\left(V - \sum_i \frac{B_i^2}{4f_i}\right)\left(1 - \sum_i \frac{P_i^2}{f_i}\right)} - \sum_i \frac{B_iP_i}{2f_i}\,, \tag{5.8}$$

where P_i is momentum conjugate to Φ^i. Now from (5.6) we have

$$V = \tau_{S0}^2 e^{-2\Phi} g_{zz}\,, f_T = \tau_{S0}^2 e^{-2\Phi} g_{tt}\,, b_T = 2\tau_{S0}^2 e^{-2\Phi} g_{tz}\,, f_\rho = \tau_{S0}^2 e^{-2\Phi} g_{\rho\rho} \tag{5.9}$$

and hence the Hamiltonian takes the form

$$H = \sqrt{\left(\frac{g_{zz}g_{tt} - g_{tz}^2}{g_{tt}}\right)\left(-\frac{P_T^2}{g_{tt}} - \frac{P_\rho^2}{g_{\rho\rho}} + e^{-2\Phi}\tau_{S0}^2\right)} - \frac{g_{tz}}{g_{tt}} P_T\,. \tag{5.10}$$

As usual the equation of motion for T that follows from (5.8) implies that $P_T = const$. On the other hand the equation of motion for ρ is equal to

$$\frac{d\rho}{d\xi} = \frac{\delta H}{\delta P_\rho} = \left(\frac{g_{tz}^2 - g_{zz}g_{tt}}{g_{tt}}\right) \frac{P_\rho}{g_{\rho\rho}\sqrt{(\ldots)}}\,. \tag{5.11}$$

If we again express P_ρ from the Hamiltonian (5.8) and use the fact that it is conserved (In a sense that $\frac{dH}{dz} = 0$) we obtain

$$P_\rho^2 = P_T)^2 + P_T^2 + \frac{L^4\tau_{S0}^2}{\alpha' g_s^2}\sinh^2\rho\,. \tag{5.12}$$

Using this expression the equation (5.11) is equal to

$$\begin{aligned}\left(\frac{d\rho}{d\xi}\right)^2 &= \left(\frac{g_{tz}^2 - g_{zz}g_{tt}}{g_{tt}}\right)^2 \frac{P_\rho^2}{g_{\rho\rho}^2(E + \frac{g_{zt}}{g_{tt}}P_T)^2} \\ &= -\frac{\alpha'^2}{L^4}\frac{\cosh^2\rho}{\sinh^2\rho} + \frac{\alpha'^4}{L^8}\frac{\cosh^4\rho}{\sinh^4\rho(E - \frac{\alpha'}{L^2}P_T)^2}\left(P_T^2 + \frac{L^4\tau_{S0}^2}{\alpha' g_s^2}\sinh^2\rho\right)\end{aligned} \tag{5.13}$$

using

$$\frac{g_{tz}^2 - g_{zz}g_{tt}}{g_{tt}g_{\rho\rho}} = -\frac{\alpha'^2}{L^4}\frac{\cosh^2\rho}{\sinh^2\rho}\,. \tag{5.14}$$

To compare the "dynamics" of S0-brane with the dynamics of D0-brane given in equation (3.17) we have to take into account the parametrization of ρ. Namely, in (3.17) ρ is the function of t that was identified with X^0 while in the equation (5.13) ρ is the function of ξ that is identified with z. On the other hand we can certainly write

$$\frac{d\rho}{d\xi}=\frac{d\rho}{dT}\frac{dT}{d\xi}=\frac{d\rho}{dT}\frac{\delta H}{\delta P_T}\,, \tag{5.15}$$

where

$$\frac{dH}{dP_T}=\frac{\alpha'}{L^2}\frac{\frac{\alpha'}{L^2}P_T+E\sinh^2\rho}{\sinh^2\rho(E-\frac{\alpha'}{L^2}P_T)}\,. \tag{5.16}$$

If we combine (5.13), (5.15) and (5.16) we get

$$\left(\frac{d\rho}{dT}\right)^2=\cosh^2\rho-\cosh^4\rho\sinh^2\rho\frac{\left[\frac{L^4}{\alpha'^2}E^2-\frac{L^4\tau_{S0}^2}{\alpha' g_s^2}\right]}{(P_T+\frac{L^2}{\alpha'}E\sinh^2\rho)^2}\,. \tag{5.17}$$

However one can check that this is exactly the same differential equation for ρ as in case of D0-brane given in (3.17) when we note that $\tau_{S0}=i\tau_0$. Let us now try to insert the solution $\cosh\rho\cos T=C$ into the equation (5.17). After some calculations we get

$$1-\frac{1}{C^2}=\frac{\sinh^4\rho\left[\frac{L^4}{\alpha'^2}E^2-\frac{L^4\tau_{S0}^2}{\alpha' g_s^2}\right]}{(P_T+\frac{L^2}{\alpha'}E\sinh^2\rho)^2}\,. \tag{5.18}$$

Since the left side is a constant it is clear that the only way how this equation is obeyed is to demand that $P_T=0$. This is in complete agreement with the previous sections since P_T is canonical conjugate to X^0. Then the equation above implies

$$E^2=C^2\frac{\alpha'\tau_{S0}^2}{g_s^2}\,. \tag{5.19}$$

From the point of view of S0-brane worldvolume theory this is perfectly consistent result since now E is real. However from the point of view of original theory where $\tau_{S0}=i\tau_0$ we obtain imaginary E which of course is expected since τ_{S0} is imaginary.

It is also interesting to study the dependence of T on ξ. In fact, using (5.8) we obtain

$$\frac{dT}{d\xi}=\frac{\delta H}{\delta P_T}=\frac{g_{zt}^2-g_{zz}g_{tt}}{g_{tt}^2}\frac{P_T}{E+\frac{g_{tz}}{g_{tt}}}-\frac{g_{tz}}{g_{tt}} \tag{5.20}$$

that for $P_T=0$ implies

$$\frac{dT}{d\xi}=-\frac{g_{tz}}{g_{tt}}=\frac{\alpha'}{L^2}\,. \tag{5.21}$$

We see that the S0-brane does not take the fixed position in time, rather the dependence of T on $\xi=z$ is in perfect agreement with the result (3.20).

On the other hand, if we insert (5.19) into (5.13) and use $P_T=0$ we obtain a differential equation for ρ in the form

$$\left(\frac{d\rho}{d\xi}\right)^2=\frac{\alpha'^2}{L^4}\left(\frac{\cosh^4\rho}{C^2\sinh^2\rho}-\frac{\cosh^2\rho}{\sinh^2\rho}\right)\,. \tag{5.22}$$

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Even if this equation can be solved explicitly we restrict ourselves to the case when $C = 1$ in order to demonstrate the main properties of given solution. For $C = 1$ the equation above reduces into

$$\frac{d\rho}{d\xi} = \frac{\alpha'}{L^2} \cosh \rho \tag{5.23}$$

that has the solution

$$\sinh \rho = \left| \tan \left(\frac{\alpha'}{L^2} \xi \right) \right| , \tag{5.24}$$

where we have chosen the integration constant in such a way that for $\xi = 0$, $\rho = 0$. The solution (5.24) describes S0-brane that many times wraps $\xi = z$ direction before it reaches its maximum value at $\xi_{max} = \frac{L^2 \pi}{2\alpha'}$ (Note that $L^2 \gg \alpha'$ in order to trust supergravity solution.). Then it again spirals down until it reaches the point $\rho = 0$ at $\xi_f = \frac{L^2}{\alpha'} \pi$.

6. Summary and conclusion

In this paper, we have studied the unphysical dS_2-branes in the covering space of the SL(2,R) WZW model, that is the AdS_3 space time, supported by NS-NS three-form flux, and observed in the T-dual set up, the emergence of the S-branes. We have been able to present a physical interpretation of the unphysical solutions with imaginary electric flux corresponding to dS_2 branes. This becomes clear in the T-dual picture, in the form of an S0-brane that arises from the time dependent tachyon condensation on an unstable D1-brane. We have also been able to show that the previously found unphysical solutions correspond, in fact, to perfect and acceptable solutions in string theory (even if the initial configurations of tachyon that corresponds to S-brane have to be fine tuned) since they arise from the singular, time dependent tachyon condensation. We have also shown that these S-branes couple to imaginary NS-NS fields, but to real R-R fields, and hence in the terminology of [4], correspond to S^- branes. We further have analyzed the time dependent tachyon condensation on non-BPS Dp-branes in general background and found out a class of time dependent singular solutions which correspond to the S^- branes. Arguments in favor of this have also been given by studying the stress tensor, that revealed the fact that indeed these S-branes couple to imaginary NS-NS fields, but to real R-R fields. There are further directions of research that one can adopt. One of them is to analyze the unphysical branes [15] in the Nappi-Witten model. The DBI action on the unphysical branes in that background have been shown to be imaginary, and hence in the present context, might correspond to some kind of Dirichlet S-branes. One can further analyze the one loop partition function for the branes in the above backgrounds, and give interpretations in the same spirit of [4]. We hope to come back to some of these issues in near future.

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